THE GEOMETRY OF CHANCE:LOTTO NUMBERS FOLLOW A PREDICTED PATTERN

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ABSTRACT: This article is based on the text "The Ludic in Game Theory"(Gianella, 2003). With mathematically formal treatment introduced in the preliminary definitions and the proof of Theorem 1, the concepts addressed result in obtaining the linear Diophantine equations which on Geometry of Chance is used to formalize the sample spaces of probabilistic events, simple combinations of n elements taken p at a time, commonly denoted by $C_{n,p}$, and combinations with repetitions of n elements taken p at a time, denoted by $Crep_{n,p}$. Introducing the idea of the frequentist view, proposed by Jacques Bernoulli, it is shown that, within the universally accepted mathematical probabilistic view of the relationship between all favorable outcomes and all possible outcomes, the result of each event follows a given pattern. The study of the set of organized and ordered patterns is introduced; accordingly, when compared to one another, the results occur with different frequencies. As predicted by the Law of Large Numbers, these patterns, if geometrically depicted, provide a simple tool with which to inspect sample spaces. Thus, the available results from lottery experiments, gathered from countries where such events take place, make up the ideal laboratory: they provide subsidies to help understand the probability of each pattern pertaining to the pattern set. Additionally, analyzing the frequencies of previous samplings provides tools for plotting strategies to forecast what might happen in the future.

KEYWORDS: Gambling; betting; pattern; template; probability; games; probability; pattern.

Introduction

Although there is a large body of literature on probability theory, such as basic references (Feller, 1976; Grimmett, 2001; Gianella, 2006), individuals' selections for lottery games are often based on birthdates, dreams and "lucky numbers," all of which are sources that are devoid of rational mathematical foundation.

Lacking modulating parameters, most people bet in lottery games the same way as their ancestors. The importance of lottery games is indicated by the data from 2011, according to which the lottery sales volume worldwide totaled US$ 262 billion (Scientific Games). Lotteries are regulated and operated by national or state/provincial governments, and a significant fraction of the proceedings goes to the state coffers, typically more than 1/3. This money is mainly used to finance activities of a social and cultural nature, education being the fore most goal for which it is used.

This article is structured as follows. Section~1 is introductory and presents the basic definitions that will be used; section~2 introduces the Brazilian Super Sena as a case

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study lottery; in section 3, the concept of a template, i.e., a betting pattern, is introduced in addition to various facts related to it; section 4 shows the reader how to improve his bets based on the facts presented on templates; and section 5 presents the conclusions.

Justification

The notion of chance dates from the Egyptian civilization. Probability theory is often found in close relationship with gambling in the studies of Cardano, Galileo, Pascal, Fermat and Huygens. From the solution of the "problem of points," the nature of gambling was first seen as a mathematical structure. The game was a mathematical model, analogous to an equation or a function. The frequentist notion, as proposed by Jacques Bernoulli in 1713 in a classical work, approximates the probability of a given event by the frequency observed when the experiment is repeated a significant number of times. With respect to the formal concept of probability, throughout mathematical history, there has been great difficulty in the choice of a model that expresses the connection between the ideal and real worlds. Through the concept of a template, games/bets are classified into patterns. The model presented here, based on the Law of Large Numbers, illustrates knowledge of the geometric organization of discrete sample spaces, which have gained from behavior patterns with preset theoretical probabilities. By deducing different behaviors, it enables the use of knowledge of the future in decision-making. The Brazilian lottery game known as Super Sena is used as our case study.

Preliminary definitions

We denote the set of natural numbers$\{0, 1, 2, \ldots\}$by $\mathbb{N}$ and the set $\{1, 2, 3, \ldots\}$by $\mathbb{N}^*$. We denote the set of real numbers by $\mathcal{R}$.

Let n and p be real numbers such that $n \geq p$. Let $M=\{a_1, a_2, \ldots, a_n\}$. The *number of combinations of the elements of M taken p by p* and represented by $C_{n,p}$ is given by

$$Cn,p= \frac{n!}{p!(n-p)!}\,.$$

Using the same n and p, the number of combinations of the elements of N taken p by p with repetition and represented by $C^{rep}_{n,p}$ is given by

$$C^{rep}_{n,p}=C_{n+p-1,p}.$$

Let A be a set and $A_1,\ldots, A_n$ sets with $n\geq 1$. We say that $(A_1,\ldots, A_n)$ is a *cover* for A if

$$A\subseteq A_1\cup A_2\cup \ldots \cup A_n.$$

Let A be a set and $(A_1,\ldots, A_n)$ with $n\geq 1$ be a cover for A. We say that $(A_1,\ldots, A_n)$ is a partition of$A$ if

$$A=A_1\cup A_2\cup \ldots \cup A_n \text{ and } A_i \cap A_j= \emptyset, \forall i,j\ 1\leq i,j \leq n.$$

Let A, $B\subseteq\mathbb{N}$ with B finite. A *coloring of A with colors of B* is a function $c : A \rightarrow B$. We say that the elements of B are colors. Let c be a coloring of A with colors from B, and let $Im(c)=\{c0, c_2,\ldots , c_k\}$.We say that *A was colored with colors* $c_0, c_2, \ldots , c_k$.

Let $E_i \subseteq A$ *such that* $c(e)=c_i\ \forall\ e \in E_i$.

We say that E_i *is colored with color* c_i or that E_i *has color* c_i.

A *finite probability space* is comprised of a finite set $\Omega \neq \emptyset$ together with a function P:$\Omega \rightarrow \mathcal{R}^+$ such that $\forall\omega \in \Omega, P(\omega) > 0$ and $\sum_{\omega\in\Omega} P\ (\omega) = 1$.

The set Ω is the *sample space*, and the function *P* is the *distribution of probabilities over*. Ω. The elements $\omega \in \Omega$ are called *basic events*. An *event* is a subset of Ω.

Let Ω be a sample space and *P* be a distribution of probabilities. We define the *probability function* or, simply, the *probability over* Ω as being the function $Pr : \wp(\Omega) \rightarrow \mathcal{R}^+$such that:

$$Pr(A) := \sum_{\omega\in A} P(\omega),\ \forall\ A \subseteq\ \Omega.$$

We note that, by definition, $Pr(\{\omega\}) = P(\omega)$, $Pr(\emptyset) = 0$ and $Pr(\Omega) = 1$.

T*rivial events* are those with a probability of 0 and 1, that is, the events $\emptyset$ and Ω, respectively. Throughout in this paper, we also denote probabilities by percentages.

The *uniform distribution over the sample space Ω* is defined by setting

$$Pr(\omega) \coloneqq 1/|\Omega|$$

For all $\omega \in \Omega$. Based on this distribution, we obtain the *uniform probability space over* Ω.

By definition, we note that, in a uniform probability space, for any event $A\ \subseteq\ \Omega,\ Pr(A) = |A|/|\Omega|$.

The definition of a uniform probability space is a formalization of the notion of "fair," as in the case of "fair dice."All of the faces of a fair die have an equal probability outcome.

Let the equation $x_1+\ x_2+\ldots+x_r=\ n$ with $x_1,\ x_2,\ \ldots,x_r$ and *n* natural numbers. This equation is a particular case of *diophantine equations*. We know from number theory that the possible number of natural solutions for this equation is $C_{n+r-1,r}$.

Let *n* and *p* be natural numbers with $n \geq p$.We say that a lottery is *p*/*n* if *p* numbers are drawn(without repetition) from the set $S_n := \{1,\ 2,\ \ldots,\ n\}$.Each subset consisting of *p* drawn numbers is called a *game* or *bet*. We denote these by *p*-uples in increasing order.

2.Case Study: Brazilian Super Sena

We begin our study with the Brazilian Super Sena, which was operated by Caixa Econômica Federal (a bank controlled by the Brazilian government)from 1995 to 2001 [4,5]. We could use any *p*/*n* lottery; however, we decided to study an existing lottery with a "large" number of drawings to compare the probabilities of certain events with the observed frequencies.

Super Sena is a 6/48 lottery; thus, the number of possible bets is given by

$$C_{48,6} = \frac{48!}{6!(48-6)!} = 12.271.512\ .$$

Moreover, because Super Sena is fair, the setΩ of possible 12.271.512 bets is a sample space; together with the function $P(\omega) \coloneqq 1/12.271.512, \forall\ \omega \in \Omega$ is a uniform probability space.

In the following, instead of considering the numbers of S_{48}, we consider each group of 10numbers as follows:

$$D_i := \{i * 10 + j \text{ tal que } 0 \leq j \leq 9\} \text{ para } 0 \leq i \leq 3 \text{ and } D_4 := \{40, \ldots, 48\}.$$

Clearly, $(D_0, D_1, D_2, D_3, D_4)$ is a partition of S_{48}.

Let B := $\{c_0,\ldots,c_4\}$ be a set of colors such that $c_0 \leq c_1 \leq c_2 \leq c_3 \leq c_4$, and let $c: S_{48} \rightarrow B$ such that $c(k) := c_i$ if and only if $k \in D_i$ $0 \leq i \leq 4$.

We can say that D_i has color c_i for $i \in \{0,\ldots, 4\}$, or intuitively, each group of 10 numbers of Super Sena has its own color.

Hereafter, let us concentrate on the colors of the bets. A *template* $T_{go,\ldots,gn}$ is an ordered(n+1) t-uple of colors $(g_0, g_1,\ldots,g_n)$ such that

$$g_0 \leq g_1 \leq \ldots \leq g_n \text{ and } g_i \in \{g_0, g_1,\ldots,g_n\}, 0 \leq i \leq n.$$

We say that n+1 is the *size* of the template. Because Super Sena is a 6/48 lottery and we have 5possible groups of 10 numbers, every template has size 6 and contains at least 1color, which appears more than once. Hereafter, we consider templates of size 6 only.

We can establish a membership relation of bets with colors in the following way. Let $(a_0, a_1, a_2, a_3, a_4, a_5)$ be a bet, and let $T_{go,\ldots,g5}$ be a template.

$$(a_0, a_1, a_2, a_3, a_4, a_5) \in G_{go,\ldots,g5} \text{ if and only if } c(a_i) = g_i, 0 \leq i \leq 5.$$

A template $G_{go,\ldots,g5}$ is said to be *monochromatic* if $g_0 = g_1 = g_2 = g_3 = = g_4 = g_5$.

The following theorem provides us with important information regarding templates.

Theorem 1. The number of possible templates of Super Sena is 210.

Proof. Let us recall that in Super Sena we draw (without replacement) 6 numbers from 5groups of 10 numbers (colors). Let us consider an arbitrary template $T_{go,\ldots,g5}$. Let x_i =the number of times that the color c_i appears in $(g_0, g_1, g_2, g_3, g_4, g_5)$, for $0 \leq i \leq 4$.Because1of the colors obligatorily appears more than once, we can write the following *diophantine* equation

$$x_0 + x_1 + x_2 + x_3 + x_4 = 6$$

That is, there is at least 1color that contributes 2 in the above sum. As presented in the section of preliminary definitions, from number theory, we have that the number of natural solutions for this equation is $C_{6+5-1,5-1} = C_{10,4} = 210$. ■

Intuitively, each of these 210 templates provides us with a “way” of betting: these “ways” of betting are given by the color of the templates. In the next section, we categorize several templates according to their colors and probabilities of occurrence.

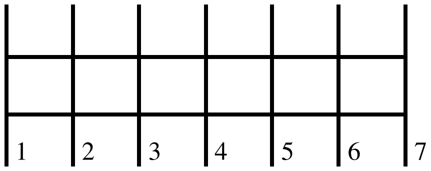
In the following, we present an alternative version of the proof of Theorem 1 through a geometric method, illustrating the idea on which this paper is focused.

Alternative proof of Theorem 1. Weagain draw (without replacement) 6 numbers from 5groups)(colors) of 10numbers. Let us consider an arbitrary template $G_{go,\ldots,g5}$. Let x_i = number of times that the color c_i appears in $(g_0, g_1, g_2, g_3, g_4, g_5)$, for $0 \leq i \leq 4$.Because1of the colors obligatorily appears more than once, we can write the following diophantine equation

$$x_0+x_1+ x_2+ x_3 + x_4 = 6$$

That is, there is at least 1 color that contributes 2 in the above sum.

Because the sum is 6, we draw 6 squares with traces.



How many traces are need to divide 5 integers?

Because there are 5 integer numbers ($x_0x_1x_2x_3$ x_4), we use 4 dividing traces to represent a solution to the equation. Thus, p=4.

How many positions to put traces in 6 spaces?

To establish the possible positions for the traces in 6 spaces, we have n=7.

How many ways (distinct or not) can be chosen to put 4 separatory traces in 7 distinct positions?

Therefore,$C^{rep}_{7,4}$=$C_{7+4-1,4}$ = 210 (Trotta, 1988). ■

2.1Categorization of Templates

In the following, let us write a table of all 210 templates. First, let us recall that a template $T_{go,\ldots,gn}$ is an ordered (n+1)-tuple of colors (g_0, g_1,…,g_n) such that $g_0 \leq g_1 \leq \ldots \leq g_n$ and $g_i \in$ {c_0,c_1,…,c_n},$0 \leq i \leq n$. In the case of Super Sena, *n* is 5, that is, colors c_0,c_1, c_2, c_3 andc_4. We present Table 1with all 210 templates in the Appendix. Table 1 was written in decreasing order of template probability.

We present some facts regarding Table 1.

The following theorem can be proved by counting the templates in Table 1, or it can be proved by computing the combinations. This gives the probability of monochromatic templates.

Theorem 2. Monochromatic templates of colors c_0 and $c_4$havea 0.0007% probability of occurring.

Proof. Let us recall that each color in {c_0,c_4}colors 9 numbers. Without loss of generality, let us consider color $c_0$and compute the probability of occurrence of template $T_{c0,\ldots,c0}$. The probability of having color $c_0$in 6 spaces is $C_{9,6}$=84.Because the probability of occurrence of the templates is uniform, the probability of occurrence of a monochromatic template is84/12.271.512 = 0.0000068,or 0.00068%,which we approximate to 0.0007%. The same argument works for c_4. ■

The following theorem is also presented for monochromatic templates.

Theorem 3. Monochromatic templates of colors $c_{1,}$ $c_{2,}$ andc_3 have probabilities of occurrence of 0.0017%.

Proof. Let us recall that each color in $\{c_{1,}\ c_{2,}\ c_3\}$colors 10 numbers. Without loss of generality, let us consider color c_1 and compute the probability of occurrence of template $T_{c1,\ldots,c1}$. The probability of having color $c_1$in 6 spaces is $C_{10,6}$=210.Because the probability of occurrence of the templates is uniform, the probability of occurrence of a monochromatic template is210/12.271.512 = 0.0000171,or 0.00171%,which we approximate to 0.0017%. The same argument works for $c_2$andc_3. ■

Although monochromatic templates have very low probabilities of occurrence, we observe from both theorems above that the probability of occurrence of monochromatic templates with colors $c_{1,}$ $c_2$andc_3(0.00171%) is more than double the probability of occurrence of monochromatic templates with colors c_0 andc_4 (0.0007%).

We present more facts regarding Table 1.

Fact 1. Each template has an exact number of combinations, and the sum of the combinations of all templates is the same as that given by the formula C48,6= 48!/(48-6)!6!.

Fact 2. Dividing the total number of outcomes favorable to each template by the total number of possible outcomes yields, a priori and exactly, the probability of each.

Fact 3. The 210 templates can be split into 39 groups with different probabilities.

Fact 4. The templates recurring most often (group 1), i.e., the templates with higher probabilities of occurrence, have little less than 3% chance, i.e., they occur 3 times every 100draws, while those from group 39, with 0.0007% chance, occur 7 times every 1,000,000 draws.

Fact 5. Each of the first 35 templates has a probability between 3% and 1%; together, they represent approximately 50%, while the remaining 175 templates make up the other 50%.

Table 1 can be rewritten so that we can obtain more information on the behavior of bets.

1. First, we write all templates with color c_0 in the first position.
2. Next, we write all templates with color $c_0$in the first and in the second position only.
3. We apply the same reasoning to the first 3 position and then to the first 4 positions, until we obtain a unique template at the end with 6 colored (c_0) positions.

4. We apply the same reasoning for color c_1: we write all templates colored with color $c_1$in the first position only; then, we write all templates with color c_1 in the first and second positions only. Applying the same reasoning for the other colors, we obtain a unique template at the end that has all of its positions colored with color c_1.
5. Use the same reasoning for colors c_2, $c_3$andc_4.
6. This way of grouping the templates according to their initial monochromatic segments is said to be *sequence by start.* We can present the following facts regarding the templates on Table 1after rewriting according to the sequence by start.

Fact 6. The templates that start with color $c_0$have a probability of occurrence of approximately 42%.

Fact 7. There are 5 templates containing (only) 1pair of the same color, and they have a probability of occurrence of 14%.

Before presenting more facts regarding Table 1, we introduce some notation for the templates.

P	-	monochromatic pair
PP	-	2 monochromatic pairs of different colors
PPP	-	3 monochromatic pairs of different colors
Q	-	monochromatic quartet
QP	-	monochromatic quartet and monochromatic pair of different color
S	-	monochromatic template
T	-	monochromatic trio
TP	-	monochromatic trio and monochromatic pair of different color
TT	-	2 monochromatic trios of different colors
V	-	monochromatic quintet

In Table 2 below, we present the (theoretical) probabilities of the above-defined configurations.

We present other facts regarding the table above.

Fact 8. There are 5 templates of type P, and they represent 14.19% of the possibilities.

Fact 9. Templates PP represent the most frequent type, with 38% probability of occurrence.

Table 2 –The frequency of certain templates

TemplateType	Number of Templates	Total Number of Combinations of the Template	Probability
P	5	1,741,500	14.19
PP	30	4,695,975	38.27
PPP	10	703,485	5.73
Q	30	484,470	3.95
QP	20	145,152	1.18
S	5	798	0.01
T	20	1,852,800	15.18
TP	60	2,498,040	20.35
TT	10	110,736	0.90
V	20	38,556	0.31
Total	210	12,271,512	100.00

3. Improvements on Betting

All of the facts that we presented regarding Table 1 give us parameters for the probabilities of the templates and for how the best work in Super Sena. We observed 963 drawings of Super Sena and made a comparison between the results obtained and the theoretical results. We briefly present these comparisons in Table 3, below.

Table 3 –A comparison of the theoretical and practical data

Template	After 963 Drawings	
	Theoretical Probability (%)	ObservedFrequency (%)
P	14.19	15.99
PP	38.27	39.56
PPP	5.73	4.88
Q	3.95	2.91
QP	1.18	0.93
S	0.01	0.00
T	15.18	13.60

Fact 10. The average of the module of the difference between the theoretical probability and the observed frequency is 0.8%.

Intuitively, Fact 9 tells us that the observed frequencies after 963 drawings are based on a sufficient number of extractions and are “very close” to the theoretical probability values.

Using the information that the theorems and the facts above provide, we can choose more frequent templates and avoid less frequent templates to improve our bets.

Conclusion

The Geometry of Chance, using Super Sena as a case study, presents the concept of a template, which is an intuitive way of representing the 6 numbers. Several template patterns were analyzed, and among them, the patterns with the highest and lowest probability of occurrence were of particular interest.

The knowledge unraveled by the Geometry of Chance is an innovative use of Combinatorics and Probability Theory, the origins of which are stated in the following.

1. In the solution by Pascal and Fermaton the "problem of points," the nature of games first started to be seen as a mathematical structure.
2. (Partial) The study undertaken by Pascal used $C_{n,p}$ and $C_{repn,p}$, which are formulas that determine these sample spaces, and are the same as that used in our study.

By presenting the mathematical structure of these experiments, i.e., the probabilistic mathematical model, the Geometry of Chance makes this type of study possible. As a main aspect, it reveals that, although all bets are equally likely, behavior patterns obey different probabilities, which can make all the difference in the concept of games, benefitting gamblers that make use of the rational information revealed by the Geometry of Chance.

GIANELLA, R. Geometria da chance: números de loterias tem um comportamento previsível. *Rev. Bras. Biom.*, São Paulo, v.31, n.4, p.582-597, 2013.

RESUMO: O artigo em questão baseia-se no texto "O Lúdico na Teoria dos jogos (Gianella, 2003). Com tratamento matematicamente formal introduzido nas definições preliminares e na demonstração do Teorema 1, os conceitos abordados resultam na obtenção das equações lineares diofantinas que na Geometria do Acaso equaciona os espaços amostrais dos eventos probabilísticos das combinações simples de n elementos tomados p de cada vez, conhecidos pela simbologia $C_{n,p}$, e das combinações com repetição de n elementos tomados p de cada vez, representados por $C_{repn,p}$ introduzindo a ideia da visão frequentista, proposta por Jacques Bernoulli, mostra que dentro da probabilidade matemática universalmente aceita da relação entre total de casos favoráveis e o total de casos possíveis, o resultado de cada evento dá-se em um determinado padrão. Apresenta o estudo do conjunto de padrões organizado e ordenado em que, comparativamente entre si, os resultados obedecem a frequências diversas. Como prevê a Lei dos Grandes Números, estes padrões geometricamente ilustrados, possibilitam, de maneira simples, acompanhar espaços amostrais. Sinaliza que os resultados disponíveis dos experimentos lotéricos, nos países onde estes eventos são realizados, é o laboratório ideal que fornece subsídios para entender a probabilidade de cada padrão pertencente ao conjunto de padrões, e ao mesmo tempo em que analisa as frequências das extrações passadas, fornece as ferramentas que possibilitam traçar estratégias para formular o que poderá acontecer no futuro.

PALAVRAS CHAVES: Jogo; aposta; padrão; gabarito;probabilidade; jogos; probabilidade; padrões de comportamento

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Appendix

Table 1 – A table with all of the templates for Super Sena (6/48), colored with colors c_0, c_1, c_2, c_3 and c_4 in decreasing order of template probability.

COLOURS CONVENTION		
C0		Yellow
C1		Blue
C2		Ash
C3		Green
C4		Rose

Number	Template						No. Combinations	Theoretical Prob.(%)	Probability Group
1	Yellow	Blue	Blue	Ash	Green	Rose	364.500	2,9703%	1
2	Yellow	Blue	Ash	Ash	Green	Rose	364.500	2,9703%	
3	Yellow	Blue	Ash	Green	Green	Rose	364.500	2,9703%	
4	Yellow	Yellow	Blue	Ash	Green	Rose	324.000	2,6403%	2
5	Yellow	Blue	Ash	Green	Rose	Rose	324.000	2,6403%	
6	Yellow	Blue	Blue	Ash	Ash	Green	182.250	1,4851%	3
7	Yellow	Blue	Blue	Ash	Green	Green	182.250	1,4851%	
8	Yellow	Blue	Ash	Ash	Green	Green	182.250	1,4851%	
9	Blue	Blue	Ash	Ash	Green	Rose	182.250	1,4851%	
10	Blue	Blue	Ash	Green	Green	Rose	182.250	1,4851%	
11	Blue	Ash	Ash	Green	Green	Rose	182.250	1,4851%	
12	Yellow	Blue	Blue	Ash	Ash	Rose	164.025	1,3366%	4
13	Yellow	Blue	Blue	Green	Green	Rose	164.025	1,3366%	
14	Yellow	Ash	Ash	Green	Green	Rose	164.025	1,3366%	
15	Yellow	Yellow	Blue	Blue	Ash	Green	162.000	1,3201%	5
16	Yellow	Yellow	Blue	Ash	Ash	Green	162.000	1,3201%	
17	Yellow	Yellow	Blue	Ash	Green	Green	162.000	1,3201%	
18	Blue	Blue	Ash	Green	Rose	Rose	162.000	1,3201%	
19	Blue	Ash	Ash	Green	Rose	Rose	162.000	1,3201%	
20	Blue	Ash	Green	Green	Rose	Rose	162.000	1,3201%	
21	Yellow	Yellow	Blue	Blue	Ash	Rose	145.800	1,1881%	6
22	Yellow	Yellow	Blue	Ash	Ash	Rose	145.800	1,1881%	
23	Yellow	Yellow	Blue	Blue	Green	Rose	145.800	1,1881%	
24	Yellow	Yellow	Ash	Ash	Green	Rose	145.800	1,1881%	
25	Yellow	Yellow	Blue	Green	Green	Rose	145.800	1,1881%	
26	Yellow	Yellow	Ash	Green	Green	Rose	145.800	1,1881%	

27							145.800	1,1881%	
28							145.800	1,1881%	
29							145.800	1,1881%	
30							145.800	1,1881%	
31							145.800	1,1881%	
32							145.800	1,1881%	
33							129.600	1,0561%	
34							129.600	1,0561%	7
35							129.600	1,0561%	
36							108.000	0,8801%	
37							108.000	0,8801%	
38							108.000	0,8801%	8
39							108.000	0,8801%	
40							108.000	0,8801%	
41							108.000	0,8801%	
42							97.200	0,7921%	
43							97.200	0,7921%	
44							97.200	0,7921%	9
45							97.200	0,7921%	
46							97.200	0,7921%	
47							97.200	0,7921%	
48							91.125	0,7426%	10
49							84.000	0,6845%	11
50							84.000	0,6845%	
51							75.600	0,6161%	
52							75.600	0,6161%	
53							75.600	0,6161%	12
54							75.600	0,6161%	
55							75.600	0,6161%	
56							75.600	0,6161%	
57							72.900	0,5941%	
58							72.900	0,5941%	
59							72.900	0,5941%	13
60							72.900	0,5941%	
61							72.900	0,5941%	
62							72.900	0,5941%	
63							58.320	0,4752%	14
64							58.320	0,4752%	

65							58.320	0,4752%	
66							54.000	0,4400%	15
67							54.000	0,4400%	
68							54.000	0,4400%	
69							54.000	0,4400%	
70							54.000	0,4400%	
71							54.000	0,4400%	
72							48.600	0,3960%	16
73							48.600	0,3960%	
74							48.600	0,3960%	
75							48.600	0,3960%	
76							48.600	0,3960%	
77							48.600	0,3960%	
78							48.600	0,3960%	
79							48.600	0,3960%	
80							48.600	0,3960%	
81							48.600	0,3960%	
82							48.600	0,3960%	
83							48.600	0,3960%	
84							43.200	0,3520%	17
85							43.200	0,3520%	
86							43.200	0,3520%	
87							43.200	0,3520%	
88							43.200	0,3520%	
89							43.200	0,3520%	
90							43.200	0,3520%	
91							43.200	0,3520%	
92							43.200	0,3520%	
93							43.200	0,3520%	
94							43.200	0,3520%	
95							43.200	0,3520%	
96							38.880	0,3168%	18
97							38.880	0,3168%	
98							38.880	0,3168%	
99							38.880	0,3168%	
100							38.880	0,3168%	
101							38.880	0,3168%	
102							37.800	0,3080%	19
103							37.800	0,3080%	
104							37.800	0,3080%	

105							37.800	0,3080%	
106							37.800	0,3080%	
107							37.800	0,3080%	
108							37.800	0,3080%	
109							37.800	0,3080%	
110							37.800	0,3080%	
111							37.800	0,3080%	
112							37.800	0,3080%	
113							37.800	0,3080%	
114							34.020	0,2772%	20
115							34.020	0,2772%	
116							34.020	0,2772%	
117							34.020	0,2772%	
118							34.020	0,2772%	
119							34.020	0,2772%	
120							30.240	0,2464%	21
121							30.240	0,2464%	
122							30.240	0,2464%	
123							30.240	0,2464%	
124							30.240	0,2464%	
125							30.240	0,2464%	
126							21.000	0,1711%	22
127							21.000	0,1711%	
128							21.000	0,1711%	
129							18.900	0,1540%	23
130							18.900	0,1540%	
131							18.900	0,1540%	
132							18.900	0,1540%	
133							18.900	0,1540%	
134							18.900	0,1540%	
135							18.900	0,1540%	
136							18.900	0,1540%	
137							18.900	0,1540%	
138							18.900	0,1540%	
139							18.900	0,1540%	
140							18.900	0,1540%	
141							17.010	0,1386%	24
142							17.010	0,1386%	
143							17.010	0,1386%	

144							14.400	0,1173%	25
145							14.400	0,1173%	
146							14.400	0,1173%	
147							12.600	0,1027%	26
148							12.600	0,1027%	
149							12.600	0,1027%	
150							12.600	0,1027%	
151							12.600	0,1027%	
152							12.600	0,1027%	
153							11.340	0,0924%	27
154							11.340	0,0924%	
155							11.340	0,0924%	
156							11.340	0,0924%	
157							11.340	0,0924%	
158							11.340	0,0924%	
159							10.080	0,0821%	28
160							10.080	0,0821%	
161							10.080	0,0821%	
162							10.080	0,0821%	
163							10.080	0,0821%	
164							10.080	0,0821%	
165							9.450	0,0770%	29
166							9.450	0,0770%	
167							9.450	0,0770%	
168							9.450	0,0770%	
169							9.450	0,0770%	
170							9.450	0,0770%	
171							7.560	0,0616%	30
172							7.560	0,0616%	
173							7.560	0,0616%	
174							7.560	0,0616%	
175							7.560	0,0616%	
176							7.560	0,0616%	
177							7.056	0,0575%	31
178							5.670	0,0462%	32
179							5.670	0,0462%	
180							5.670	0,0462%	
181							5.670	0,0462%	
182							5.670	0,0462%	

183							5.670	0,0462%	
184							4.536	0,0370%	33
185							4.536	0,0370%	
186							2.520	0,0205%	34
187							2.520	0,0205%	
188							2.520	0,0205%	
189							2.520	0,0205%	
190							2.520	0,0205%	
191							2.520	0,0205%	
192							2.268	0,0185%	35
193							2.268	0,0185%	
194							2.268	0,0185%	
195							2.268	0,0185%	
196							2.268	0,0185%	
197							2.268	0,0185%	
198							1.260	0,0103%	36
199							1.260	0,0103%	
200							1.260	0,0103%	
201							1.260	0,0103%	
202							1.260	0,0103%	
203							1.260	0,0103%	
204							1.134	0,0092%	37
205							1.134	0,0092%	
206							210	0,0017%	38
207							210	0,0017%	
208							210	0,0017%	
209							84	0,0007%	39
210							84	0,0007%	
							12.271.512	**100,00%**	